LESSON 4. 4b

Factoring Polynomials by Grouping

Today you will:

- Use the grouping technique to factor polynomials

- Practice using English to describe math processes and equations

Core Vocabulary:

- Factor by grouping, p. 181

- Quadratic form, p. 181

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- Factor by grouping, p. 181

  - Group pairs of terms that have common monomial factor.

  - Only works for some polynomials.

  - The pattern:

  $$ra + rb + sa + sb = r(a + b) + s(a + b)$$
  $$= (r + s)(a + b)$$

- Example: $x^3 + 2x^2 + 3x + 6 = x^2(x + 2) + 3(x + 2)$
  $$= (x^2 + 3)(x + 2)$$

Factor $z^3 + 5z^2 - 4z - 20$ completely.

SOLUTION

$$z^3 + 5z^2 - 4z - 20 = z^2(z + 5) - 4(z + 5) \qquad \text{Factor by grouping.}$$

$$= (z^2 - 4)(z + 5) \qquad \text{Distributive Property}$$

$$= (z - 2)(z + 2)(z + 5) \qquad \text{Difference of Two Squares Pattern}$$

Factor $x^3 + 4x^2 - x - 4$ completely.

SOLUTION

$$x^3 + 4x^2 - x - 4 = x^2(x + 4) - (x + 4)$$ Factor by grouping.

$$= (x^2 - 1)(x + 4)$$ Distributive Property

$$= (x - 1)(x + 1)(x + 4)$$ Difference of Two Squares Pattern

Factor $3y^3 + y^2 + 9y + 3$ completely.

SOLUTION

$3y^3 + y^2 + 9y + 3 = y^2(3y + 1) + 3(3y + 1)$ Factor by grouping.

$= (y^2 + 3)(3y + 1)$ Distributive Property

Core Vocabulary:

- Quadratic form, p. 181

  - An expression in the form $au^2 + bu + c$ where u is an algebraic expression.

  - Example: $x^4 + 2x^2 + 1$

    - Perfect square trinomial: $a^2 + 2ab + b^2 = (a+b)^2$, $a = x^2$, $b = 1$
    - $x^4 + 2x^2 + 1 = (x^2)^2 + 2x^2 + (1)^2 = (x^2 + 1)(x^2 + 1)$

  - Example: $9x^4 - 16$

    - Looks like difference of squares $a^2 - b^2 = (a-b)(a+b)$, $a = 3x^2$, $b = 4$
    - $9x^4 - 16 = (3x^2)^2 - (4)^2 = (3x^2 - 4)(3x^2 + 4)$

Factor (a) $16x^4 - 81$ and (b) $3p^8 + 15p^5 + 18p^2$ completely.

The expression $16x^4 - 81$ is in quadratic form because it can be written as $u^2 - 81$ where $u = 4x^2$.

SOLUTION

a. $16x^4 - 81 = (4x^2)^2 - 9^2$ Write as $a^2 - b^2$.

$\qquad\qquad\quad = (4x^2 + 9)(4x^2 - 9)$ Difference of Two Squares Pattern

$\qquad\qquad\quad = (4x^2 + 9)(2x - 3)(2x + 3)$ Difference of Two Squares Pattern

b. $3p^8 + 15p^5 + 18p^2 = 3p^2(p^6 + 5p^3 + 6)$ Factor common monomial.

$\qquad\qquad\qquad\qquad\quad = 3p^2(p^3 + 3)(p^3 + 2)$ Factor trinomial in quadratic form.

Homework

Pg 184, #23-38